A Correlation for Pressure Drop in Two-Phase Cocurrent Flow in Packed Beds

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A correlation has been developed from considerations of single-phase flow behavior to predict pressure drop across packed beds for two-phase cocurrent flow. The correlation does not require one of the assumptions made in previous correlations, and thus it was not necessary to relate empirically any dimensionless groups through the use of experimental two-phase flow data. The only empiricism involved in the use of this correlation is that required in correlating single-phase pressure drops through packed beds, an art well developed in the literature. The correlation as developed here is at least as accurate as previous correlations in this area and may be more reliable when used for two-phase systems not previously studied experimentally.

The use of catalytic bed processes to carry out reactions between gas and liquid phases is becoming quite prevalent in the chemical and petroleum industries. From a design standpoint, it is desirable to have a method available for calculation of pressure drop across the catalyst bed, particularly if compression equipment is required for recycle of the gas phase.

Although a number of investigators (*1* to *3*) have obtained experimental pressure drops across packed beds for two-phase cocurrent flow systems, the majority of data available are for air-water systems at ambient conditions. Correlations developed from these data are largely empirical; consequently, it is uncertain as to whether they are useful when applied to systems which differ considerably in physical properties or phase flow rates from those studied experimentally.

The purpose of this investigation was to obtain a pressure drop correlation for two-phase cocurrent flow in packed beds, based on consideration of single-phase flow behavior in these systems, which would apply to all types of two-phase systems and would require a minimum of experimental data to evaluate any empirical parameters necessary to the correlation.

PREVIOUS STUDIES ON TWO-PHASE PRESSURE DROP CORRELATIONS

Open Pipe

Lockhart and Martinelli (*4*) presented one of the first important correlations for two-phase pressure drop in open horizontal tubes. The basic assumptions used in the development of this correlation were:

1. Static pressure drop for the liquid phase must equal the static pressure drop for the gaseous phase regardless of the flow pattern, as long as an appreciable radial static pressure difference does not exist.

2. The volume occupied by the liquid phase plus the volume occupied by the gas phase at any instant must equal the total volume of the pipe.

On the basis of these assumptions, several dimensionless groups were developed to characterize the two-phase pressure drop in terms of the single-phase pressure drops which would exist if each phase flowed separately through the pipe:

$$\phi_L^2 = \delta'_{LGF}/\delta_{LF} \quad (1)$$

$$\phi_G^2 = \delta'_{LGF}/\delta_{GF} \quad (2)$$

$$X^2 = \delta_{LF}/\delta_{GF} \quad (3)$$

It was then postulated that ϕ_L and ϕ_G were unique functions of X, and experimental data were used to develop these functions. The results were presented in terms of curves for ϕ_L and ϕ_G vs. X for each of the four flow mechanisms possible in the systems (that is, turbulent gas-turbulent liquid, turbulent gas-viscous liquid, viscous gas-turbulent liquid, and viscous gas-viscous liquid). The requirement of a separate curve for each flow mechanism is evidently due to the different dependence of δ_{LF} and δ_{GF} on the Reynolds number for viscous and turbulent flow in open pipes.

Packed Beds

Larkins, White, and Jeffrey (*5*), beginning with the same assumptions as Lockhart and Martinelli, developed an analogous correlation to apply to two-phase pressure drop in packed beds. The dimensionless group used to characterize the two-phase pressure drop in this system was

$$F(X) = \delta'_{LGF}/(\delta_{LF} + \delta_{GF}) \quad (4)$$

When this group was plotted vs. X for an extensive set of experimental data, a single curve was sufficient to represent all the data. This might be expected, since the dependence of δ_{LF} and δ_{GF} on the modified Reynolds number in a packed bed does not show the sharp break in the transitional region between viscous and turbulent flow found in open pipes, but instead follows a smooth curve between fully developed viscous flow and fully developed turbulent flow.

To apply the correlation to downflow beds, Larkins and co-workers used a head correction term based on the average fluid density in the bed:

$$\delta'_{LGT} = \delta'_{LGF} - R_L\,\rho_L - (1 - R_L)\,\rho_G \quad (5)$$

where R_L, the fraction of void volume of the bed occupied by the liquid phase, was found from an experimentally determined dependence of R_L on X:

$$\log_{10} R_L = -0.744 + 0.525 \log_{10} X - 0.109\,(\log_{10} X)^2 \quad (6)$$

The two-phase pressure drop equation developed from their experimental data was

$$\log_{10}\left[\frac{\delta'_{LGF}}{\delta_{LF} + \delta_{GF}}\right] = \frac{0.416}{(\log_{10} X)^2 + 0.666} \quad (7)$$

BASIS FOR DEVELOPMENT OF A NEW CORRELATION

The purpose of the present work was twofold.

1. Although it appeared that the two basic assumptions originally used by Lockhart and Martinelli and later by

Larkins and co-workers were sound, it was desirable to eliminate the experimental determination of the relationships between the various dimensionless groups. If it were possible to derive these relationships through mathematical development, the need for experimental determination would be eliminated. Ideally, it might be possible to develop a correlation which would require no empirical evaluation of parameters from experimental two-phase pressure drop data. If such a correlation could be developed and could closely represent the available experimental data, it could be used in design work for two-phase systems differing greatly in physical properties or flow conditions from systems studied experimentally, with a high degree of confidence in the reliability of the results.

2. Larkins and co-workers developed their two-phase correlation using a homogenous flow model, although the resulting dimensionless groups involve only properties obtained from single-phase flow of liquid and gas through the bed. The model used in the present work assumes that two continuous phases are present in the bed. Since this model differs substantially in concept from that used by Larkins, it is interesting to examine the pressure drop predicted by the two models and to compare the results with experimental data.

DEVELOPMENT OF CORRELATION

The development of the present correlation requires three basic assumptions in addition to those stated by Lockhart and Martinelli:

1. Two-phase flow through the packed bed is such that the liquid phase flows uniformly over the packing surface, and both liquid and gas phases are continuous.
2. A pressure drop equation is available which adequately predicts the frictional pressure drop through the bed in terms of bed characteristics, fluid properties, and fluid flow rates when there is single-phase flow in the system, that is, when the liquid or gas phase flows through the bed in the absence of the other phase.
3. The pressure drop equation adequately predicts the frictional pressure drop through the bed for either phase when there is two-phase flow, provided that the presence of the other flowing phase is taken into consideration.

The initial requirement is a single-phase pressure drop relation which satisfies assumption 2. For this purpose, the equation proposed by Ergun and Orning (6) was selected. This equation, developed from basic considerations of single-phase flow through a fixed bed, can be expressed in the form

$$\delta_F = \frac{4\rho u^2}{g_c D}\left[\beta + 4\alpha\left(\frac{\mu}{Du\rho}\right)\right] \tag{8}$$

where the equivalent channel diameter D is defined as

$$D = \frac{4\epsilon}{(1-\epsilon)S_0} \tag{9}$$

The average fluid velocity u can be related to the superficial fluid velocity $\bar{u}$ by

$$u = \frac{\bar{u}}{\epsilon} \tag{10}$$

Substitution of Equations (9) and (10) in Equation (8) gives

$$\delta_F = \frac{\rho\bar{u}^2(1-\epsilon)S_0}{g_c\,\epsilon^3}\left[\beta + \frac{\alpha\mu(1-\epsilon)S_0}{\bar{u}\rho}\right] \tag{11}$$

This equation should adequately represent single-phase pressure drop for any packed bed if the constants α and β are properly chosen.

Now assume a two-phase flow system in which the liquid phase flows uniformly over the packing surface and both liquid and gas phases are continuous. For the liquid phase, assume the frictional losses can be based on the packing surface area and the average velocity of the liquid with respect to this surface. Also assume a liquid volume factor θ_L which characterizes the effective fraction of the void volume in the bed occupied by the liquid phase. In this case

$$u_L = \frac{\bar{u}_L}{\theta_L\,\epsilon} \tag{12}$$

$$D_L = \frac{4\,\theta_L\,\epsilon}{(1-\epsilon)S_0} \tag{13}$$

Therefore, the frictional pressure drop through the liquid phase is

$$\delta'_{LF} = \frac{\rho_L\,\bar{u}_L^2\,(1-\epsilon)S_0}{\theta_L^3\,\epsilon^3\,g_c}\left[\beta + \frac{\alpha\mu_L(1-\epsilon)S_0}{\bar{u}_L\,\rho_L}\right] \tag{14}$$

Dividing by the value of δ_{LF} obtained for single-phase liquid flow from Equation (11), one gets

$$\frac{\delta'_{LF}}{\delta_{LF}} = \frac{1}{\theta_L^3} \tag{15}$$

To represent truly the frictional energy loss through the liquid phase, the term δ'_{LF} must be the result of energy transfer to the liquid from the gas and the dissipation at the packing surface. The above development must therefore be considered oversimplified in that it does not directly consider any drag effect exerted upon the liquid phase by the flowing gas phase, and there is no consideration of the nature of the interaction between the liquid phase and the packing surface, such as if the liquid phase wets the packing. Therefore, Equation (14) may be viewed as an approximation of the true frictional pressure loss through the liquid phase. Comparison of the results predicted by the final correlation with actual experimental data should indicate the degree of approximation involved.

For the gas phase, assume that the frictional losses can be based on the wetted surface area of the packing and the average velocity of the gas phase with respect to the average velocity of the liquid phase. Assume also a gas volume factor θ_G, analogous to θ_L, and defined so that

$$\theta_L + \theta_G = 1 \tag{16}$$

In this case, the velocity of the gas phase relative to that of the liquid phase is

$$u_G = \frac{\bar{u}_G}{\theta_G\epsilon} - \frac{\bar{u}_L}{\theta_L\epsilon} = \frac{\bar{u}_G\,Y}{\theta_G\epsilon} \tag{17}$$

where

$$Y = 1 - \frac{\theta_G\,\bar{u}_L}{\theta_L\,\bar{u}_G} \tag{18}$$

Since the liquid is assumed to flow uniformly over the surface of the packing, the number of packing particles is the same as in single-phase flow, but the effective packing volume and surface area have changed because of the presence of the uniform liquid film. If, for convenience, a spherical packing is chosen, it is easily shown that

$$D_G = \frac{4\,\theta_G\,\epsilon}{(1-\epsilon)S_0\,Z^2} \tag{19}$$

where

$$Z = \left(1 + \frac{\theta_L\epsilon}{1-\epsilon}\right)^{1/3} \tag{20}$$

Thus

$$\delta'_{GF} = \frac{\rho_G \bar{u}_G^2 (1-\epsilon) S_0 Y^2 Z^2}{\theta_G^3 \epsilon^3 g_c} \left[\beta + \frac{\alpha \mu_G (1-\epsilon) S_0 Z^2}{\bar{u}_G \rho_G Y} \right] \tag{21}$$

Dividing by the value of δ_{GF} obtained for single-phase gas flow from Equation (11), one obtains

$$\frac{\delta'_{GF}}{\delta_{GF}} = \frac{H}{\theta_G^3} \tag{22}$$

where

$$H = YZ^2 \left\{ \frac{\beta Y + \dfrac{\alpha \mu_G (1-\epsilon) S_0 Z^2}{\bar{u}_G \rho_G}}{\beta + \dfrac{\alpha \mu_G (1-\epsilon) S_0}{\bar{u}_G \rho_G}} \right\} \tag{23}$$

The basic relations required for the correlation developed in this work are given by Equations (15) and (22). The resulting two-phase pressure drop correlation developed from these relations has been named the *total equivalence method* (TEM) to emphasize one of the major assumptions made in its derivation; namely, that each phase can be considered continuous throughout the bed and that the total pressure drop through each phase is identical. This is in contrast with the use of a homogenous flow model by Larkins and co-workers.

APPLICATION OF THE CORRELATION TO DOWNFLOW BEDS

The total pressure drop through the liquid phase is given by the combinations of friction loss and fluid head terms for the liquid phase:

$$\delta'_{LT} = \delta'_{LF} - \rho_L = \frac{\delta_{LF}}{\theta_L^3} - \rho_L \tag{24}$$

The corresponding expression for the gas phase is

$$\delta'_{GT} = \delta'_{GF} - \rho_G = \frac{\delta_{GF} H}{\theta_G^3} - \rho_G \tag{25}$$

It is assumed that the total pressure drop through each phase is the same; therefore

$$\delta'_{LGT} = \frac{\delta_{LF}}{\theta_L^3} - \rho_L = \frac{\delta_{GF} H}{\theta_G^3} - \rho_G \tag{26}$$

Since θ_L and θ_G are related by Equation (16), they can be eliminated (with the exception of their appearance in the definition of H) to give the final relation for TEM as applied to downflow beds:

$$\left[\frac{\delta_{LF}}{\delta'_{LGT} + \rho_L} \right]^{1/3} + \left[\frac{\delta_{GF} H}{\delta'_{LGT} + \rho_G} \right]^{1/3} = 1 \tag{27}$$

Actually, Equation (27) is not particularly useful in the form presented. To solve for the value of δ'_{LGT}, the following procedure can be used:

1. Assume a value of θ_L.
2. Calculate values for θ_G and H.
3. Calculate a new value of θ_L from the relation

$$\theta_L = \frac{1}{\left(\dfrac{\delta_{GF}}{\delta_{LF}} \right)^{1/3} \left[H + \theta_G^3 \left(\dfrac{\rho_L - \rho_G}{\delta_{GF}} \right) \right]^{1/3} + 1} \tag{28}$$

4. Repeat steps 2 and 3 until a constant value of θ_L is obtained (generally 3 to 6 iterations).
5. Use Equation (26) to calculate δ'_{LGT}.

Therefore, once values have been obtained for δ_{LF} and δ_{GF} from any reliable single-phase pressure drop correlation, a value can be calculated by TEM for the total two-phase pressure drop without the use of any additional empirical equations.

It should be noted that the term θ_L as used in the development of this correlation is not necessarily equal to the true liquid saturation R_L; it is merely an internal parameter of the final correlation, and its actual physical significance may include effects other than the true liquid saturation.

TABLE 1. RANGE OF CONDITIONS FOR LITERATURE DATA

Data source	Dodds et al.	Wen et al.
Packing type (Size in inches)	Raschig rings (2) Berl saddles (1, 1½) Intalox saddles (1, 1½)	Raschig rings (¼, ½, 1) Berl saddles (½) Intalox saddles (½, 1)
Liquid phase compositions	(1) Water (2) 2.5 N Na_2CO_3 soln.	$CaCl_2$ soln.
Flow rate, lb./(sq. ft.)(min.)	70.8 to 623.3	39.5 to 276.9
Density, lb./cu. ft.	62.1 to 69.7*	86.7 to 87.1*
Viscosity, centipoise	0.807 to 0.823*	7.2 to 7.5*
Gas phase compositions	Air	Air
Flow rate, lb./(sq. ft.)(min.)	7.2 to 30.7	2.0 to 23.7
Density, lb./cu. ft.	0.072*	0.074*
Viscosity, centipoise	0.020*	0.018*
No. of data points		
Single phase liquid flow	0	0
Single-phase gas flow	27	54
Two-phase flow	343	378
Data source	Larkins nonhydrocarbon	Larkins hydrocarbon
Packing type (Size in inches)	Raschig rings (⅜) Cylinders (⅛) Spheres (⅜)	Cylinders (⅛) Spheres (0.012)
Liquid phase composition	(1) Water (2) 2.5% methocel soln. (3) 0.5% methocel soln. (4) 0.033% soap soln. (5) Ethylene glycol	(1) Kerosene (2) Hexane (3) Lube oil
Flow rate, lb./(sq. ft.)(min.)	71.5 to 2730	8.2 to 250
Density, lb./cu. ft.	62.4 to 69.5	41.2 to 53.2
Viscosity, centipoise	0.76 to 17.6	0.33 to 38.8
Gas phase compositions	Air	(1) Natural gas (2) Carbon dioxide
Flow rate, lb./(sq. ft.)(min.)	3.5 to 101.4	0.011 to 7.1
Density, lb./cu. ft.	0.08 to 0.45	0.043 to 0.112
Viscosity, centipoise	0.017 to 0.020	0.012 to 0.015
No. of data points		
Single-phase liquid flow	156	0
Single-phase gas flow	27	0
Two-phase flow	570	65

* Estimated.

COMPARISON OF CORRELATION WITH LITERATURE DATA

The TEM was used to predict two-phase pressure drop in downflow packed beds for 1,356 data points found in the literature. On the basis that the value of H is generally close to unity, the correlation was also tested in a modified form in which the value of H was assumed to be unity. In essence, this modified form of the correlation neglects the effect of liquid velocity and the change in surface-to-volume ratio for the packing due to the liquid film when calculations are made for the pressure drop through the gas phase. The correlation developed by Larkins, White, and Jeffrey (LWJ) was also included in the comparison study.

For the data of Larkins (*2*) on nonhydrocarbon systems, the correlations developed from single-phase flow data were used to calculate the required single-phase pressure drops. For the data on hydrocarbon systems, the reported values for single-phase pressure drop were used after the appropriate corrections mentioned by Larkins were made for average fluid density.

To use the remainder of the literature data (*1, 3*), it was necessary to develop single-phase pressure drop correlations on the basis of the data presented for single-phase gas flow (no data were obtained for single-phase liquid flow). By using estimated bed porosities and fluid properties (none was given in the references), a single-phase pressure drop equation of the form given by Equation (11) was fitted to the available single-phase gas flow data by adjusting the values of α and β. A separate equation of this type was developed for each packing material used. It was then assumed that each equation was also applicable for single-phase flow of liquid through the same packing material. Although these estimations undoubtedly introduced a considerable amount of error into the calculations, these data were included since they covered a range of flow conditions and packing materials not considered by Larkins.

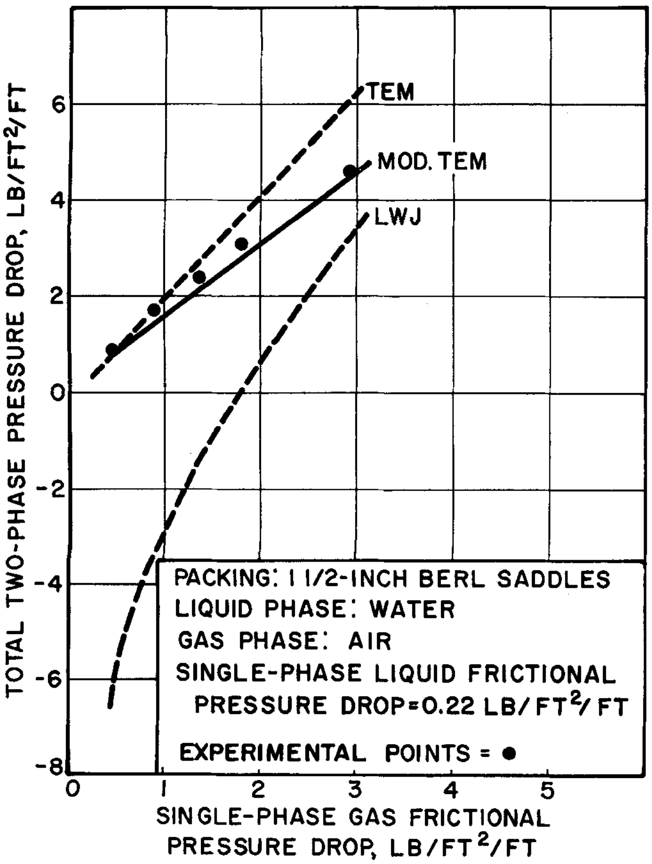

Fig. 1. Typical comparison of correlations with data of Dodds.

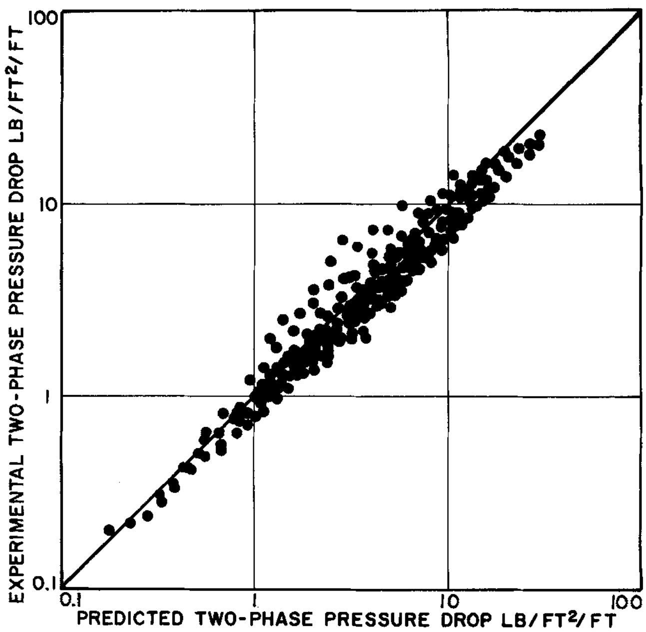

Fig. 2. Overall comparisons of TEM with data of Dodds.

Table 1 summarizes the experimental data found in the literature and the range of variables covered.

Figure 1 shows a typical comparison of the correlations with the data of Dodds and co-workers. The modified TEM generally gives the best overall fit with the experimental data. TEM tends to give high values for cases where both liquid and gas rates are high, whereas LWJ greatly underestimates pressure drop at the lowest gas rates. In fact, LWJ generally predicts a pressure rise across the bed at low gas rates, although none of the experimental data indicated this to be true. Figure 2 summarizes the overall comparison of the experimental data with predictions by TEM.

Comparisons of the correlation with the experimental data of Wen and co-workers indicate a less satisfactory comparison that was found with the data of Dodds. Of the three correlations, TEM appears to give the best esti-

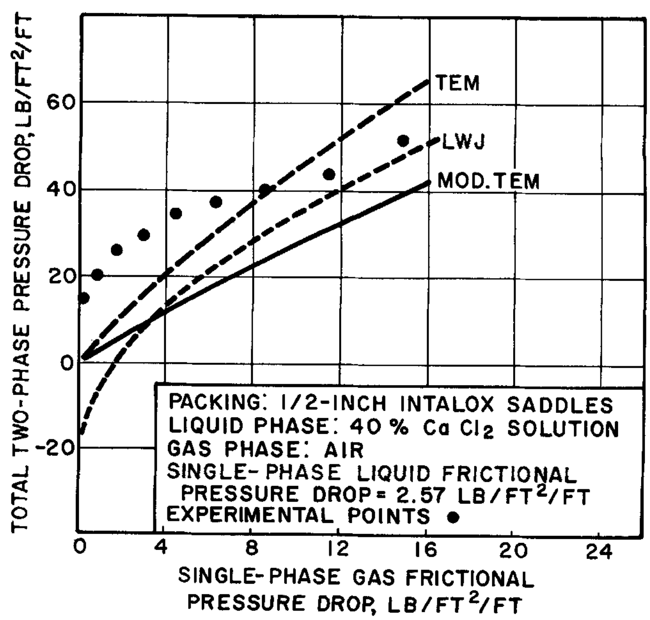

Fig. 3. Typical comparison of correlations with data of Wen.

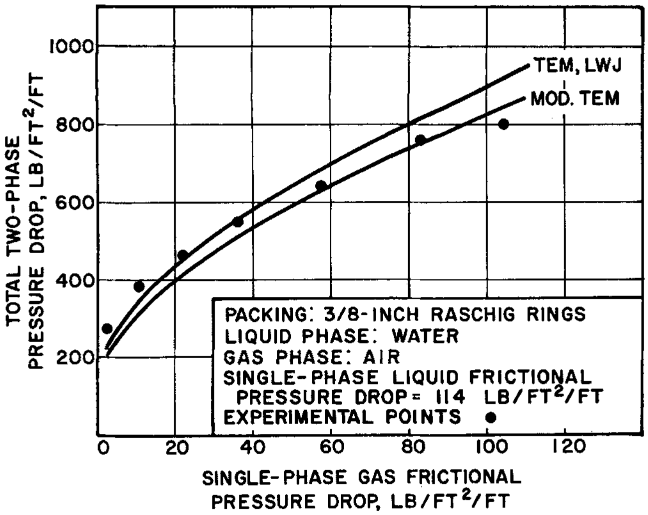

Fig. 4. Typical comparison of correlations with nonhydrocarbon data of Larkins.

mates of two-phase pressure drop but generally predicts low values at the lowest gas rates and high values at higher gas rates. Again, LWJ indicates a pressure rise across the bed at the lowest gas rates, although this was not indicated experimentally. In general, all correlations are better for the larger packing sizes than for the smaller ones.

A typical comparison of correlations with the experimental data of Wen is shown in Figure 3.

The failure of the correlations to represent the data of Wen closely must lie in part on the extensive estimations required to apply the correlations to this set of data. In particular, it may be a poor approximation to assume that a single-phase pressure drop equation based on air flow is also suitable to describe the flow of a relatively viscous salt solution over the same packing. Although the same estimations were required to include the data of Dodds in this study, the assumptions in the latter case may be more valid owing to the relatively low viscosity of the liquid phase and the use of larger packing sizes. This explanation is strengthened by some observations of Larkins which will be pointed out later.

Figure 4 shows a typical comparison between correlations and the experimental data of Larkins on nonhydrocarbon systems. In general, all three correlations give very good agreement with the data except for those systems in which significant foaming occurs. For nonfoaming systems, the deviations between predicted and experimental two-phase pressure drop are roughly equivalent to those between calculated and experimental single-phase pressure drop, indicating that the latter calculation is the major source of error in this comparison.

It is interesting to note that TEM and the modified TEM, whose developments are completely independent of these data, are as accurate as LWJ which is empirically based on this particular set of data.

The overall comparison of TEM with the nonhydrocarbon data of Larkins is shown in Figure 5.

Figures 6 and 7 show typical comparisons of the correlations with the hydrocarbon data obtained by Larkins. The agreement between correlations and data is much less satisfactory for these systems than for the nonhydrocarbon systems, even when data are excluded for those systems in which significant foam was observed. For the data on kerosene-natural gas shown in Figure 6, all correlations give moderately low values at the higher gas rates. At lower gas rates, TEM and the modified TEM agree well with the experimental data, whereas LWJ is somewhat low and often indicates a pressure rise across the bed at the lowest gas rates. Figure 7, which illustrates the comparison for a lube oil-natural gas system, shows all correlations to be consistently low at the lower gas rates.

It should be noted that the comparisons with the hydrocarbon systems of Larkins involve additional approximations which were not necessary in the comparisons for the nonhydrocarbon data. Larkins points out that the single-phase pressure drops for the hydrocarbon data are not based on actual experimental single-phase flow data in these systems but are obtained from equations developed previously for the nonhydrocarbon systems. Thus, the single-phase pressure drop equation developed for flow over ⅜-in. stoneware spheres was used directly for flow over 3-mm. glass beads with no adjustment of constants. This undoubtedly introduces an error into the calculations, but it is a necessary assumption in the absence

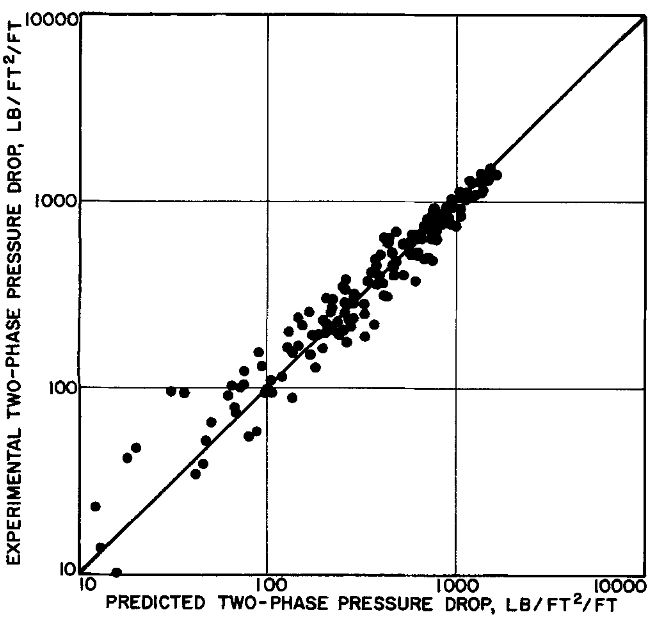

Fig. 5. Overall comparison of TEM with nonhydrocarbon data of Larkins.

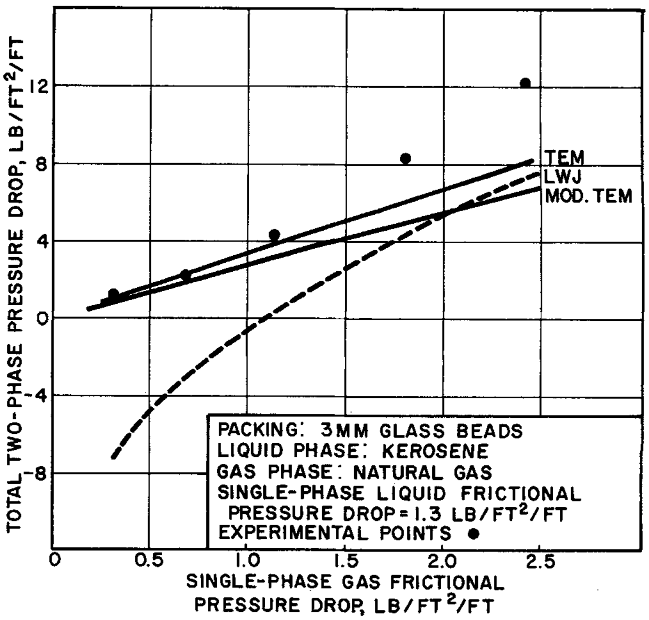

Fig. 6. Typical comparison of correlations with hydrocarbon data of Larkins on a kerosene-natural gas system.

of experimental single-phase flow data for the systems under consideration.

It is interesting to note that the comparisons shown in Figure 7 are quite similar to those shown in Figure 3 for the data of Wen. In both cases, the systems involve relatively viscous liquids, small packing sizes, and the absence of data for single-phase flow of liquid over the packing under consideration. This suggests that this particular combination of circumstances can introduce considerable error into the calculations. This conclusion is strengthened by the observation of Larkins that his experimental liquid saturation data indicated an upward shift with an increase in viscosity and with a decrease in packing size which is not accounted for by the variables in Equation (6). Since the liquid volume factor used in TEM is a function of essentially the same variables as the true liquid saturation, it may also not account properly for liquid viscosity and packing size. It must therefore be emphasized that all correlations may give very conservative values for the flow of relatively viscous liquids over small packing sizes, particularly at low flow rates. If the correlations are to be used under these circumstances, the results should be judged accordingly. It is strongly suggested that data should be obtained for single-phase flow of liquid in these cases.

On the basis of the above comparisons, it appears that all three correlations adequately predict two-phase pressure drop across packed beds for cocurrent, downflow systems except in certain instances where inadequate data are available to describe single-phase flow behavior in the system or in cases where significant foaming occurs. It is believed that the accuracy of all correlations would have been improved if better single-phase correlations had been available (particularly for liquid flow) and if it had not been necessary to estimate bed porosity and fluid properties for a portion of the literature data. Accuracy of the correlations might also have been improved if the compressibility of the gas phase had been taken into consideration by including kinetic energy terms in the equations for those cases where large pressure drops existed.

A most interesting result of the comparisons of the correlations with experimental data is that all correlations are usually in agreement. Since two completely different flow models were involved in the development of these correlations, substantial differences might have been expected in predicted values under certain conditions. However, with the exception of the pressure rise given by the correlation of Larkins and co-workers at low flow rates, there is no basic disagreement among the correlati

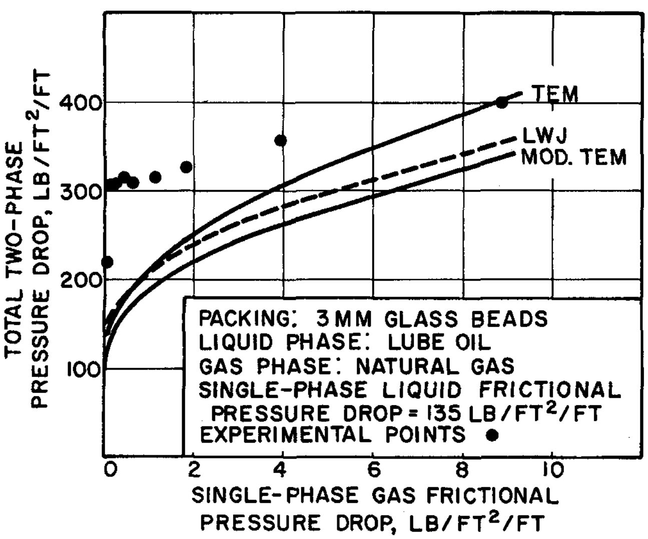

Fig. 7. Typical comparison of correlations with hydrocarbon data of Larkins on a lube oil-natural gas system.

Another interesting result is that TEM is in relatively good agreement with the majority of the experimental data even though the drag effect exerted on the liquid phase by the flowing gas phase was not directly considered in the development of this correlation. This implies that the drag effect was relatively unimportant in relation to other effects considered, or that other assumptions were made which compensated for this omission.

Although the assumption of two continuous phases might appear to limit the use of TEM to certain restricted flow regimes, it is felt that this restriction is not as serious as it first appears. Lockhart and Martinelli made essentially the same assumption in the development of correlations for two-phase flow in pipes; yet their correlations successfully predict pressure drop even when other flow conditions exist (7). Evidence for the applicability of TEM to other flow regimes is given by the nonhydrocarbon data of Larkins. These data cover a variety of two-phase flow conditions, and the correlation appears equally good for all except when significant foaming occurs in the system.

Since TEM and its modified form appear to be equally satisfactory in representing the experimental data, the modified form ($H = 1$) is to be preferred because of its simplicity.

SUMMARY

A correlation, TEM, has been developed from considerations of single-phase flow behavior to predict two-phase pressure drop for cocurrent flow in packed beds. The final correlation does not require one of the assumptions made in previous correlations, and thus it was not necessary to relate empirically any dimensionless groups through the use of experimental two-phase data. The only empiricism involved in the use of this correlation is that required in correlating single-phase pressure drops through packed beds, an art well developed in the published literature. The correlation as developed is at least as accurate as previous correlations in this area and may be more reliable when used on two-phase systems not previously studied experimentally.

All correlations considered may tend to give conservative values in instances where the liquid phase is relatively viscous and the packing size is small. It is strongly recommended that in these instances particular care should be taken to obtain single-phase flow data for the liquid before the correlations are applied.

The form of TEM is

$$\left(\frac{\delta_{LF}}{\delta'_{LGT} + \rho_L}\right)^{1/3} + \left(\frac{\delta_{GF}}{\delta'_{LGT} + \rho_G}\right)^{1/3} = 1 \quad (30)$$

ACKNOWLEDGMENT

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NOTATION

D = equivalent channel diameter, ft.

$F(X)$ = function defined by Equation (4)

g_c = gravitational conversion constant, (lb.$_m$) (ft.)/(lb.$_f$) (sec.^{2})

H = factor defined by Equation (23)

R = fraction of void volume occupied by one fluid phase, dimensionless

S_0 = surface area of packing particle, sq.ft. surface area/cu.ft. particle volume

u = linear velocity of fluid phase, ft./sec.

$\bar{u}$ = superficial linear velocity of fluid phase, ft./sec.
X = dimensionless group defined by Equation (3)
Y = factor defined by Equation (18)
Z = factor defined by Equation (20)

Greek Letters

α, β = constants in Equation (11)
δ = pressure drop per unit length calculated for single-phase flow, $(\text{lb.}_f)(\text{sq.ft.})/\text{ft.}$
δ' = pressure drop per unit length calculated for two-phase flow, $\text{lb.}_f(\text{sq.ft.})/\text{ft.}$
ϵ = void fraction of packed bed volume, dimensionless
θ = saturation factor, dimensionless
μ = fluid viscosity, $\text{lb.}_m/(\text{ft.})(\text{sec.})$
ρ = fluid density, $\text{lb.}_m/\text{cu.ft.}$
ϕ = dimensionless groups defined by Equations (1) and (2)

Subscripts

F = frictional pressure drop
G = gas phase
L = liquid phase
LG = two phase
T = total pressure drop

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Transition and Film Boiling from Horizontal Strips

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Measurements of heat flux and surface temperature fluctuations are reported for transition and film boiling of Freon 113 from flattened horizontal stainless steel tubes. The strip width is found to have a definite effect upon the heat flux in the film boiling regime. Photographic measurements indicate the absence of any appreciable long-range periodicities, from which it is inferred that random turbulent effects are quite important. Also, a spectrum of bubble diameters, frequencies, and spacings is observed. On the other hand, some short-time spatial periodicities were observed, and the theoretical values of the parameters fell well within the experimental range. Based upon the photographic and heat transfer measurements, a recommended expression for the minimum heat flux was developed.

Unlike nucleate boiling, transition and film boiling of a pool of liquid from a horizontal heating surface may be independent of the nature of the heating surface, so that a purely hydrodynamic theory is possible. The basic theory is due to Zuber and Tribus (*1*), who assume that the frequency and spacing of vapor release points are governed by the stability of the interface between the vapor and the liquid. The spacing of vapor release points is determined by the wave number for Taylor instability of a liquid layer of semi-infinite extent lying over a lighter fluid which is also of semi-infinite extent. For the maximum heat flux in transition boiling, the maximum allowable relative velocity of the vapor jets leaving the surface and the liquid "spikes" returning to the surface is considered to be determined by Helmholtz instability of the respective interfaces. It is assumed that nowhere in the transition boiling region does the liquid come into contact with the heating surface.

The theory was subsequently modified by Zuber (*2*, *3*), by Berenson (*4*, *5*), and by Lienhard and Wong (*6*). Alternative models, based upon the assumption of liquid-solid contact in transition boiling, have also been proposed (*7*). Experimental measurements of bubble patterns and flux–ΔT curves have been made by Westwater et al. from tubes and flat plates (*8* to *10*) which support some aspects of the Zuber theory. A detailed review is given in reference *11*.

EXPERIMENTAL EQUIPMENT

The boiling element consisted of a partially flattened ½ in. (O.D.) stainless steel tube 4.75 in. long (Figure 1). Flatness was checked visually by means of a straight edge. The tube was completely covered with ⅛ in. of epoxy resin with the exception of a heating strip surface. Nevertheless, a few bubbles were generated at minute pinholes in the epoxy, but were kept out of the field of view by deflector fins along each side of the heating surface. These fins also served to reduce flow toward the heating surface from the sides, which can frequently be troublesome in boiling from strips (*12*). Strip widths of both ½ and ¼ in. were used, the ¼-in. strip meeting the requirement of a single straight row of bubbles. To study the dependence of heat flux upon the width of the

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